Why the Vagueness Argument is Unsound 3K

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Considerations of vagueness suggest to some that composition is unrestricted. Central to this line of thought is the intimate connection between composition and number - when composition occurs the number of things increases. However, the connection is not clear enough, especially if vagueness is semantic indeterminacy. In this paper I argue that Theodore Sider's vagueness argument for unrestricted composition fails because the sub-argument from vague composition to a vague numerical sentence fails.

1 Introduction

Thinking about how simples need to be arranged to compose something encourages universalist intuitions because it is hard to see where to draw a non-arbitrary line between classes of simples that do not compose anything and classes that do. Imagine all the classes of simples on a continuum with classes of simples that seem obviously to not strength of this arbitrariness intuition is proportional to the degree of similarity, in respects relevant to composition, we allow adjacent classes to exhibit. The more similar adjacent classes are, the more arbitrary any specific line becomes. Since any line we draw is arbitrary, there are no sharp cut-offs. And every class of simples either definitely composes something or definitely does not - it is not as though some class of simples sort of composes something and sort of doesn't. Whether a class has a fusion is like whether a number is prime - it does not admit borderline cases. The trouble is, this little argument entails that if *any* class of simples composes something (say, the simples that compose the screen or paper on which you are reading this) then *every* class of simples, no matter how far apart or gerrymandered (six from Big Ben, three from my right earlobe, and one from the dark side of the moon) compose something.

This is a version of Theodore Sider's vagueness argument for unrestricted composition ('the vagueness argument').[1] In this paper I do two things. First, I argue that Theodore Sider's vagueness argument for unrestricted composition fails because the sub-argument from vague composition to a vague numerical sentence fails. Second, I provide a higher-order vagueness argument for unrestricted composition which apparently succeeds without appeal to the impossibility of a vague numerical sentence. The higher-order vagueness argument seems to work, except that it admits of a parody argument for unrestricted baldness. The parody shows that both the vagueness and higher-order vagueness arguments are unsound.

In §2 I explain the vagueness argument. In §3 I reconstruct Sider's only argument (the 'numerical sentence argument') for the vagueness argument's main premise. In §4 I argue that the numerical sentence argument fails and consequently, that the vagueness

[1]Sider builds his vagueness argument from raw materials found in David Lewis's proto-vagueness argument Lewis (1986) 211ff.

argument fails.

2 The Vagueness Argument

The vagueness argument goes as follows:[2]

> P1 If not every class has a fusion there must be a pair of cases [of composition] connected by a continuous series such that in one, composition occurs, but in the other, composition does not occur.
>
> P2 In no continuous series is there a sharp cut-off in whether composition occurs.[3]
>
> P3 In any case of composition, either composition definitely occurs, or composition definitely does not occur.
>
> Therefore, every class has a fusion.

A *case of composition* is a class of objects such that it is an open question whether they compose anything. A *continuous series* is a sorites series of cases of composition. A *sharp cut-off* in a continuous series would be a location between two members of the series M1 and M2 such that the members of M1 definitely compose something and the members of M2 definitely do not compose anything.

There is a lot to say about the vagueness argument.[4] In the next two sections I focus only on P3.

[2] Sider (2001), 120-132.

[3] In discussing sharp cut-offs I will use the phrase 'there cannot be a sharp cut-off between x and y' to mean 'there cannot be sharp cut-off *in a sorites sequence* between x and y' because the implicit reference to such a sequence seems obvious enough.

[4] For further discussion see Hawley (2002), Hawley (2004), Koslicki (2003), Sider (2003), Smith (2006), and Korman (MS).

3 THE NUMERICAL SENTENCE

> in question, but it would be indeterminate whether to include another entity: the fusion of the class. Now surely if P3 can be violated, then it could be violated in a 'finite' world, a world with only finitely many concrete objects. **That would mean that** some *numerical sentence*-a sentence asserting that there are exactly *n* concrete objects, for some finite *n*-would be indeterminate. But aside from the predicate 'concrete', which is non-vague, numerical sentences contain only logical vocabulary, and logical vocabulary, I say, can never be a source of vagueness. [Thus, P3] [my **emphasis**].[7]

Put in premise-conclusion form, Sider's argument runs

> A1 Suppose it is possible that it is vague whether a class of objects has a fusion.
>
> A2 That it is possible that it is vague whether a class of objects has a fusion **means** that it is possible that some numerical sentence is vague.
>
> A3 It is possible that some numerical sentence is vague only if it is possible that logical vocabulary is vague.
>
> A4 It is not possible that logical vocabulary is vague.[8]
>
> Thus, P3.[9]

Since vagueness is properly understood as semantic indeterminacy, the first part of A2 is making a semantic claim about a *sentence*. That is, the first part of A2 should be read as expressing the proposition that it is possible that some compositional sentence is vague. Thus understood, A2 can be restated as:

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[7] Sider (2001), 127.

[8] This premise can be resisted by fans of relative identity. For the argument for A4 assumes that numerical sentences (sentences that can be stated using only quantifiers, variables, and identity) are perfectly intelligible and not further analyzable in terms of sortal identity predicates like 'is the same dog as'. The fans of relative identity deny that such sentences are perfectly intelligible without further analysis into sortal identity predicates, and so could complain that numerical sentences *are* vague because they admit precisifications into different propositions about distinct sortal identity relations. For problems with relative identity see Hawthorne (2003) and Rea (2003). For further defense of A4, especially defense of the view that existence cannot be vague, see Sider (2003).

[9] Although Sider takes the possibility of vague composition to suffice for the negation of P3, this assumes a further premise - that if it is not possible that it is vague whether a class of objects has a fusion then in every possible case of composition either composition definitely occurs or composition definitely does not occur. I accept this further premise and flag it only for the sake of completeness.

A2’ That it is possible that some compositional sentence is vague **means** that it is possible that some numerical sentence is vague.

The problem is that A2’ is obviously false, and so an uncharitable reading of Sider. That it is possible that some compositional sentence is vague means something rather different from the proposition that it is possible that some numerical sentence is vague.[10] But we often use instances of the phrase schema ‘x means that y’ to mean something weaker than ‘x is synonymous with y’. I think this is the case here. Instead of uncharitably reading A2 as a synonymy claim like A2’, we should read it as the weaker entailment claim:[11]

A2* That it is possible that some compositional sentence is vague **entails** that it is possible that some numerical sentence is vague.[12]

In the next section I show that A2* lacks support.

4 A2* Lacks Support

Suppose with Sider that vagueness is semantic indeterminacy. Why might we be inclined to accept A2*? Presumably because of the tight connection between a compositional sentence and its corresponding numerical sentence. For instance, for some *n*, the compositional sentence

[10] I am not assuming a hyperintensional notion of meaning here either. As I will make clear later, the

CS

'There are exactly n simples and there are no composites.'[13]

entails its corresponding numerical sentence, NS, 'there are exactly n concrete things'. If we restrict its quantifiers to concreta, NS can be stated using only quantifiers, variables, truth-functional connectives, identity, and parentheses:[14]

NS

'$\exists x_1 \exists x_2 ... \exists x_n [x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge ... \wedge x_{n-1} \neq x_n \wedge \forall y (y = x_1 \vee y = x_2 ... \vee y = x_n)]$.'

So, we might be inclined to accept A2* because CS entails NS for every value of n.

But care is required here. We all agree that CS entails NS - that it is necessary that if CS is *true* then so is NS. But A2* makes the further claim that it is necessary that if CS is *vague* then so is NS. So far we have only seen only that CS entails NS - we have not yet seen why it is necessary that if CS is *vague* then NS is. Presumably, there is some sort of unspecified connection implicit between the first claim - that CS entails NS - and the second - that it is necessary that if CS is *vague* then so is NS. That is, the entailment relation we all obviously see and accept is supposed to (somehow!) cause us to accept the corresponding vagueness claim.

There are a few ways this might go.

We might (implicitly) think that linguistic vagueness transfers over entailment. If that is what we think, then it immediately follows from the fact that CS entails NS that it

[13] Formally, in order for CS to entail NS we must add the following conjunct to CS: 'and something is concrete iff it is simple or composite'.

[14] Instead of restricting its quantifiers to concreta we could let its quantifiers range unrestrictedly and write NS as '$\exists x_1 \exists x_2 ... \exists x_n [Cx_1 \wedge Cx_2 \wedge ... \wedge Cx_n \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge ... \wedge x_{n-1} \neq x_n \wedge \forall y (Cy \rightarrow (y = x_1 \vee y = x_2 ... \vee y = x_n))]$.' But since I am granting the assumption that 'is concrete' is non-vague, restricting the quantifiers simplifies the numerical sentence without misconstruing Sider's argument. It is worth pointing out here, that one could also resist the numerical sentence argument by denying that 'is concrete' is non-vague, perhaps citing (contentious!) examples of borderline cases of concreteness like persons or properties.

is necessary that if CS is *vague* then so is NS. The problem with this way of arriving at A2* is that linguistic vagueness *does not* transfer over entailment. Suppose 'Bill is bald' is linguistically vague because Bill is a paradigm case of borderline baldness. 'Bill is bald' entails '2 is prime' but '2 is prime' is not linguistically vague. So linguistic vagueness does not transfer over entailment. So the vagueness claim - A2* - does not "fall out of" the entailment claim, as we perhaps (implicitly) thought it did. So it is possible that CS entails NS, that CS is vague, and that NS is not.[15]

We might instead think that linguistic vagueness transfers over equivalence. Then the thought would be that since CS *is equivalent to* NS, if CS were vague then NS would be vague as well.[16] There are two ways to read the equivalence claim: hyperintensional and intensional equivalence. Since CS is not hyperintensionally equivalent to NS, we can ignore this reading. The claim, then, is that since CS is intensionally equivalent to NS, linguistic vagueness transfers over intensional equivalence, and NS cannot be vague, CS cannot be vague. But CS is not intensionally equivalent to NS. NS is true and CS is false in W: in W there are exactly n-1 simples and there is exactly one composite and something is concrete iff it is simple or composite. So A2* is not supported by the transference of linguistic vagueness over equivalence, because compositional sentences are not equivalent to the numerical sentences they entail (e.g., CS is not equivalent to NS).[17]

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[15] David Barnett (MS) argues for an inference rule which one might employ at this point to try to get linguistic vagueness to transfer over entailment. The inference rule is
Clearly, if p, then q, it is vague whether $p \vdash$ either it is vague whether q or it is clearly the case that q (pp. 3)
Because this rule allows that q Becau itencwhs33ntn.(ul-355)(be)-s rule tnguisticofr'38.8msT'664 Tf 249

I forsee the following objection.[18] The numerical sentence argument does not require that CS and NS are intensionally equivalent. Rather, it requires only the following weaker equivalence (along with the transference of linguistic vagueness over equivalence):

CS is true iff NS is true and there are exactly *n* simples

Although this equivalence holds, if *this* (or any other equivalence which employs the predicates 'simple' or 'composite' as part of the numerical sentence) is the equivalence to which the numerical sentence argument appeals, then the numerical sentence argument fails because 'is simple' and 'is composite' *can* be vague.[19]

So far we've seen that linguistic vagueness does not transfer over entailment, that CS and NS are not equivalent, and that appeal to a weaker equivalence (like the one above) undercuts the numerical sentence argument.[20] Perhaps linguistic vagueness nonetheless transfers over a non-equivalence relation stronger than entailment - analytic entailment, where sentence S1 analytically entails S2 iff it is analytically true that if S1 is true then S2

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ciple for generating lots of worlds at which an instance of CS is false and the instance of NS it entails is true, regardless of the numerical combination of simples-composites we pick: Let *s*, *c*, *n*, and *l* be positive integers. Let CS_n be any compositional sentence of the form 'there are exactly *s* simples and there are exactly *c* composites' where $s+c=n$. Let NS_n be any numerical sentence of the form 'there are exactly *n* concrete things'. Then (although for any value of *n*, CS_n entails NS_n) for any value of *n*, NS_n does not entail CS_n. For, for any value of *n*, NS_n is true and CS_n is false at every world where (i) there are $s+l$ simples and $c-l$ composites or (ii) there are $s-l$ simples and $c+l$ composites. And there are going to be lots of both kinds of worlds.

[18] Thanks to AS. For further discussion of related objections see **Appendix**.

[19] For discussion of simples see Markosian (1998b), Markosian (1998a), Markosian (2004a), McDaniel (2003), Markosian (2004b), McDaniel (2007a), and McDaniel (2007b).

[20] Another reply to the above argument, that the numerical sentence argument fails because CS and NS are not equivalent, might go as follows. CS is not the sentence that matters, and so is not the sort of sentence the proponent of the vagueness argument needs to be equivalent to NS. Instead, the sort of compositional sentence she needs to be equivalent to NS is the sentence CS*, 'There is some *y* that the *x*s compose.'; so all this talk of CS is a red herring. But it is not a red herring because CS* is obviously not equivalent to NS because CS* does not even entail NS (CS* is consistent with there being exactly *n*+100 concrete things). Of course, we could adjust CS* so that it entails NS, i.e., we could replace it with CS**, 'There is exactly one *y* that the *x*s compose and there are exactly *n*-1 *x*s and nothing has *y* as a part and something is concrete iff it is simple or composite.' But NS does not entail CS** for the same reason NS does not entail CS. So switching to a compositional sentence like CS** also fails to repair the numerical sentence argument.

O2 It is not possible that it is ontically indeterminate how many (concrete) things there are.

O3 So it is not possible that composition is ontically vague.

The first (obvious) cost of this reply is that it accepts the coherence of ontic vagueness. This cost will, all by itself, ward off many who would otherwise employ it in the service of unrestricted composition. The second cost is that appeal to the semantic non-vagueness of numerical sentences does not entail O2, and consequently, an appeal solely to the semantic non-vagueness of numerical sentences does not deliver O2. For, suppose we wanted to use the semantic non-vagueness of numerical sentences to argue for O2. Then we would have to demonstrate the thesis that necessarily, if it is ontically indeterminate how many things there are then some numerical sentence is semantically vague. But that thesis is false; ontic count indeterminacy does not entail the semantic vagueness of a numerical sentence for two reasons.

First, consider a languageless world w_1 where it is ontically indeterminate whether there are exactly eight things or whether there are exactly nine things. The number of things in w_1 is ontically indeterminate between eight and nine and no numerical sentence is semantically vague in w_1 because there are no sentences in w_1. So ontic count indeterminacy does not entail the semantic vagueness of any numerical sentence. So the impossibility of ontic count indeterminacy cannot be got merely by appeal to the impossibility of semantically vague numerical *sentences*.[26]

[26]One might object that this argument fails to appreciate the in/at distinction with respect to worldly truth, and claim that a sentence can be true (or vague) *at* even a languageless world, that some numerical sentence *will* be vague *at* w_1 even though it is not vague *in* w_1, and that it is impossible that any numerical sentence be vague *at* a world. This objection, however, requires weakening entailment to capture the sentences that must be true *at* the world in which the entailing proposition is true, in addition to the sentences that must be true *in* the world in which the entailing proposition is true. For, the objection succeeds only if the proposition *it is ontically indeterminate whether there are 8 or 9 things* is true in w_1 and entails that some numerical sentence is vague *at* but not *in* w_1. So this objection succeeds only if the following is correct:

Second, consider a languagefull world w_2 where it is ontically indeterminate whether there are exactly eight things or whether there are exactly nine things. That the number of things in w_2 is ontically indeterminate between eight and nine does not entail that in w_2 some numerical sentence is semantically vague. Instead, that the number of things in w_2 is ontically indeterminate between eight and nine entails that in w_2 *either* (i) some numerical sentence is semantically vague (e.g., 'there are exactly eight things' and 'there are exactly nine things' are semantically vague) *or* (ii) every numerical sentence is perfectly precise and false (e.g., 'there are exactly eight things' and 'there are exactly nine things' are both false but perfectly precise).[27] The inference from ontic vagueness to semantic vagueness is bad because ontic count indeterminacy can obtain perfectly well alongside the precise *falsity* of every numerical sentence. In fine, ontic count indeterminacy does not entail that numerical sentences are vague because (i) it could be ontically indeterminate how many things there are and there be no numerical sentences and (ii) it could be ontically indeterminate how many things there are and every numerical sentence be precisely false.

Because the defense of P3 falls far short of anything conclusive, I conclude that the vagueness argument for unrestricted composition fails.

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proposition *p* entails sentence S1 iff S1 is true in *or at* every world in which *p* is true. Weakening entailment in this way may be too high a cost to save the vagueness argument for unrestricted composition. But even if it is not too high a cost, the ontic reply fails even in the case whether the world where it is ontically indeterminate how many things there are contains all the sentences we need, as the second reply shows.

[27]Suppose numerical sentences can also specify whether there are *not* exactly *n* things. Then the second option is really (ii') every numerical sentence is either precisely false (e.g., 'there are exactly eight things') or precisely true ('there are not exactly eight things'). Thanks to MR here.

5 Appendix

Another way to cast the numerical sentence argument is by adding names to the sentences in order to secure their equivalence. There are two problems with this recasting.

First, names can be vague. So adding them to a numerical sentence allows it to be vague. In that case, its equivalence with a compositional sentence fails to show that the compositional sentence cannot be vague even if linguistic vagueness transfers over equivalence. Further, some possible compositional sentences can contain vague names, and so will be equivalent only to numerical sentences that likewise contain vague names.

But suppose we can fix this by adding only non-vague names to the sentences. There are still compositional sentences such that any numerical sentences to which they are equivalent are possibly vague. For example, we might think that the following sentences are equivalent, where 'the *x*s' rigidly designates simples x_1, x_2, ..., x_{n-1}:

CS1 'The *x*s are simple, there are exactly *n*-1 *x*s, there is exactly one subplurality of the *x*s, the *y*s, that compose anything and they compose exactly one thing, *a* (and something is concrete iff it is simple or composite, and nothing is simple and composite).'

NS1 'There are exactly *n* concrete things and every concrete thing, except for *a*, is identical to one of the *x*s.'

But these sentences are not equivalent because NS1 does not entail CS1. *W* is world where NS1 is true and CS1 is false: in *W* NS1 is true but a different subplurality of the *x*s, the *z*s, compose *a*. *W* is possible provided mereological essentialism is false. So the following sentence *is* equivalent to CS1:

NS1’ NS1 and mereological essentialism is true

But of course, because NS1’ *can* be vague (its right conjunct employs non-logical mereological vocabulary). So adding names to the respective sentences fails to repair the numerical sentence argument because (i) names can be vague and (ii) even if only non-vague names are added, there are compositional sentences (like CS1) such that the only numerical sentences to which they are equivalent are possibly vague.

REFERENCES

Barnes, E. and Williams, J. R. G. (forthcoming). A Theory of Metaphysical Indeterminacy. In Zimmerman and Bennett.

Barnett, D. Vague entailment. *Manuscript*.

Donnelly, M. (2009). Mereological Vagueness and Existential Vagueness. *Synthese*, 168:53–79.

Eklund, M. (2005). What Vagueness Consists In. *Philosophical Studies*, 125:27–60.

Hawley, K. (2002). Vagueness and Existence. *Proceedings of the Aristotelian Society*, 102:125–140.

Hawley, K. (2004). Borderline Simple or Extremely Simple. *MOnist*, 87:385–404.

Hawthorne, J. (2003). Identity. In Loux, M. J. and Zimmerman, D. W., editors, *Oxford Handbook of Metaphysics*, pages 99–130. Oxford University Press, Oxford.

Koslicki, K. (2003). The Crooked Path from Vagueness to Four-Dimensionalism. *Philosophical Studies*, 114:107–34.

Lewis, D. (1986). *On the Plurality of Worlds*. Blackwell, Oxford.

Lewis, D. (1988). Vague identity: Evans misunderstood. *Analysis*, 48:128–130.

Markosian, N. (1998a). Brutal composition. *Philosophical Studies*, 92:211–49.

Markosian, N. (1998b). Simples. *Australasian Journal of Philosophy*, 76:213–26.

Markosian, N. (2004a). Simples, stuff, and simple people. *The Monist*, 87:405–28.

Markosian, N. (2004b). Soc it to me? reply to mcdaniel on maxcon simples. *Australasian Journal of Philosophy*, 82:332–40.

McDaniel, K. (2003). Against maxcon simples. *Australasian Journal of Philosophy*, 81:265–75.

McDaniel, K. (2007a). Brutal simples. In Zimmerman, D. W., editor, *Oxford Studies in Metaphysics*, volume 3. Oxford University Press, Oxford.

McDaniel, K. (2007b). Extended simples. *Philosophical Studies*, 133:131–41.

Merricks, T. (2005). Composition and Vagueness. *Mind*, 104:615–37.

Rea, M. (2003). Relative Identity and the Trinity. *Philosophia Christi*, 5:431–446.

Sider, T. (2001). *Four-Dimensionalism*. Clarendon, Oxford.

Sider, T. (2003). Against vague existence. *Philosophical Studies*, 114:135–46.

Sider, T. (2010). *Logic for Philosophy*. Oxford University Press, Oxford.

Smith, D. (2006). The Vagueness Argument for Mereological Universalism. *Pacific Philosophical Quarterly*, 87:357–368.

Smith, N. (2005). A plea for things that aren't quite all there or is there a problem about vague composition and vague existence. *Journal of Philosophy*, 102:381–421.

Smith, N. J. J. (2009). *Vagueness and Degress of Truth*. OUP, Oxford.

van Inwagen, P. (1990). *Material Beings*. Cornell University Press, Ithaca, NY.

Williamson, T. (1994). *Vagueness*. Routledge, London.

Williamson, T. (1999). On the Structure of Higher-Order Vagueness. *Mind*, 108: 429:127–143.

Wright, C. (1975). On the coherence of vague predicates. *Synthese*, 30:325–65.

Zimmerman, D. W. and Bennett, K., editors. *Oxford Studies in Metaphysics vol 6*. Oxford University Press (forthcoming), Oxford.